# CS 325I- Computer Networks I: Routing Algorithms 

Professor Patrick Traynor 10/I/I3<br>Lecture 13

## Reminders

- The due date for Homework 2 was moved to Thursday.
- Reason:Allow you to attend today's lecture.

Project 2 is still due in one week.

- Absolutely no extensions will be given.


## Last Time

- Subnets provide granularity for address assignment and ease management.
- What is I92.I68.8.0? I92.I68.32.0?
- What is NAT? DHCP?
- What are some security issues associated with ICMP messages?


## Chapter 4: Network Layer

- 4. I Introduction
- 4.2 Virtual circuit and datagram networks
- 4.3 What's inside a router
- 4.4 IP: Internet Protocol
- Datagram format
- IPv4 addressing
- ICMP
- IPv6
- 4.5 Routing algorithms
- Link state
- Distance Vector
- Hierarchical routing
- 4.6 Routing in the Internet
- RIP
- OSPF
- BGP
- 4.7 Broadcast and multicast routing


## Interplay between routing and forwarding



## Graph abstraction



Graph: $G=(N, E)$

$$
N=\text { set of routers }=\{u, v, w, x, y, z\}
$$

$E=$ set of links $=\{(u, v),(u, x),(v, x),(v, w),(x, w),(x, y),(w, y),(w, z),(y, z)\}$
Aside: Graph abstraction is useful in other network contexts
Example: P2P, where $N$ is set of peers and $E$ is set of TCP connections

## Graph abstraction: costs



- $c\left(x, x^{\prime}\right)=$ cost of link ( $x, x^{\prime}$ )
- e.g., $c(w, z)=5$
- cost could always be I, or inversely related to bandwidth, or inversely related to congestion

Cost of path $\left(x_{1}, x_{2}, x_{3}, \ldots, x_{p}\right)=c\left(x_{1}, x_{2}\right)+c\left(x_{2}, x_{3}\right)+\ldots+c\left(x_{p-1}, x_{p}\right)$

Question:What's the least-cost path between u and z ?

Routing algorithm: algorithm that finds least-cost path

## What are the costs?

- We will speak very generally about the idea of "link cost". Some potential examples include:
- Bandwidth/Speed
- Physical Length
- Monetary Cost
- Policy Configurations


## Routing Algorithm classification

Global or decentralized information?

## Global:

- all routers have complete topology, link cost info
- "link state" algorithms

Decentralized:

- router knows physicallyconnected neighbors, link costs to neighbors
- iterative process of computation, exchange of info with neighbors


## Static or dynamic?

## Static:

- routes change slowly over time


## Dynamic:

- routes change more quickly
- periodic update
- in response to link cost changes


## Load Sensitive or Insensitive

-Respond to traffic conditions

- "distance vector" algorithms


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## A Link-State Routing Algorithm

Dijkstra's algorithm

- net topology, link costs known to all nodes
- accomplished via "link state broadcast"
- all nodes have same info
- computes least cost paths from one node ('source") to all other nodes
- gives forwarding table for that node
- iterative: after k iterations, know least cost path to $k$ dest.'s


## Notation:

- $c(x, y)$ : link cost from node $x$ to $y$; $=\infty$ if not direct neighbors
- $D(v)$ : current value of cost of path from source to dest. v
- $\mathrm{p}(\mathrm{v})$ : predecessor node along path from source to $v$
- $\mathrm{N}^{\prime}$ : set of nodes whose least cost path definitively known


## Dijsktra's Algorithm

1 Initialization:
$2 \mathrm{~N}^{\prime}=\{\mathrm{u}\}$
3 for all nodes $v$
4 if $v$ adjacent to $u$
5 then $D(v)=c(u, v)$
6 else $D(v)=\infty$

## Loop

find $w$ not in $N$ such that $D(w)$ is a minimum
11 update $\mathrm{D}(\mathrm{v})$ for all v adjacent to w and not in $\mathrm{N}^{\prime}$ :
12
13
14
15 until all nodes in $\mathbf{N}^{\prime}$

## Notation:

- $c(x, y)$ : link cost from node $x$ to $y$; $=\infty$ if not direct neighbors
- $\mathrm{D}(\mathrm{v})$ : current value of cost of path from source to dest. v
- $p(\mathrm{v})$ : predecessor node along path from source to $v$
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## Dijkstra's algorithm: example

| Step | $N^{\prime}$ | $D(v), \mathrm{p}(\mathrm{v})$ | $\mathrm{D}(\mathrm{w}), \mathrm{p}(\mathrm{w})$ | $\mathrm{D}(\mathrm{x}), \mathrm{p}(\mathrm{x})$ | $\mathrm{D}(\mathrm{y}), \mathrm{p}(\mathrm{y})$ | $\mathrm{D}(\mathrm{z}), \mathrm{p}(\mathrm{z})$ |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0 | u | $2, \mathrm{u}$ | $5, \mathrm{u}$ | $1, \mathrm{u}$ | $\infty$ | $\infty$ |
| 1 |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |



## Dijkstra's algorithm: example (2)

Resulting shortest-path tree from u:


Resulting forwarding table in u:

| destination | link |
| ---: | :--- |
| $v$ | $(u, v)$ |
| $x$ | $(u, x)$ |
| $y$ | $(u, x)$ |
| $w$ | $(u, x)$ |
| $z$ | $(u, x)$ |

## Dijkstra's algorithm, discussion

Algorithm complexity: n nodes

- each iteration: need to check all nodes, w, not in N
- $n(n+1) / 2$ comparisons: $O\left(n^{2}\right)$
- more efficient implementations possible: O(nlogn)

Oscillations possible:

- e.g., link cost $=$ amount of carried traffic

initially

given these costs, find new routing.... resulting in new costs

given these costs, find new routing.... resulting in new costs

given these costs, find new routing.... resulting in new costs


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## Distance Vector Algorithm

## Bellman-Ford Equation (dynamic programming)

Define
$d_{x}(y):=$ cost of least-cost path from $x$ to $y$
Then:
$d_{x}(y)=\min _{v}\left\{c(x, v)+d_{v}(y)\right\}$
cost from neighbor $v$ to destination $y$
cost to neighbor $v$
min taken over all neighbors v of x

## Bellman-Ford example

Clearly, $\mathrm{d}_{\mathrm{v}}(\mathrm{z})=5, \mathrm{~d}_{\mathrm{x}}(\mathrm{z})=3, \mathrm{~d}_{\mathrm{w}}(\mathrm{z})=3$


B-F equation says:

$$
\begin{aligned}
& d_{u}(z)=\min \left\{c(u, v)+d_{v}(z),\right. \\
& c(u, x)+d_{x}(z), \\
&\left.c(u, w)+d_{w}(z)\right\} \\
&= \min \{2+5, \\
& l+3,
\end{aligned}
$$

Node that achieves minimum is next

$$
5+3\}=4
$$

hop in shortest path $\rightarrow$ forwarding table

## Distance Vector Algorithm

- $D_{x}(y)=$ estimate of least cost from $x$ to $y$
- Node $x$ knows cost to each neighbor v: $c(x, v)$
- Node $x$ maintains distance vector $D_{x}=\left[D_{x}(y): y \in N\right]$
- Node x also maintains its neighbors' distance vectors
- For each neighbor $v, x$ maintains

$$
D_{v}=\left[D_{v}(y): y \in N\right]
$$

## Distance vector algorithm (4)

## Basic idea:

- Each node periodically sends its own distance vector estimate to neighbors
- When a node $x$ receives new DV estimate from neighbor, it updates its own DV using B-F equation:
$D_{x}(y) \leftarrow \min _{v}\left\{c(x, v)+D_{v}(y)\right\} \quad$ for each node $y \in N$
- Under natural conditions, the estimate $D_{x}(y)$ converge to the actual least cost $d_{x}(y)$


## Distance Vector Algorithm (5)

Iterative, asynchronous: each local iteration caused by:

- local link cost change
- DV update message from neighbor


## Distributed:

- each node notifies neighbors only when its DV changes
- neighbors then notify their neighbors if necessary


## Each node:

Wait for (change in local link cost or msg from neighbor)
recompute estimates

if $D V$ to any dest has changed, notify neighbors

$$
\begin{aligned}
D_{x}(y) & =\min \left\{c(x, y)+D_{y}(y), c(x, z)+D_{z}(y)\right\} \\
& =\min \{2+0,7+1\}=2
\end{aligned}
$$

node $x$ table

node $y$ table
$D_{x}(z)=\min \{c(x, y)+$

$$
\begin{aligned}
& \left.D_{y}(z), c(x, z)+D_{z}(z)\right\} \\
& \quad=\min \{2+1,7+0\}=3
\end{aligned}
$$

node $z$ table



$$
\begin{aligned}
D_{x}(y) & =\min \left\{c(x, y)+D_{y}(y), c(x, z)+D_{z}(y)\right\} \\
& =\min \{2+0,7+I\}=2
\end{aligned}
$$

$$
\begin{aligned}
& D_{x}(z)=\min \{c(x, y)+ \\
& \left.\quad D_{y}(z), c(x, z)+D_{z}(z)\right\} \\
& \quad=\min \{2+1,7+0\}=3
\end{aligned}
$$

node $x$ table

node y table
node $z$ table


## Distance Vector: link cost changes

## Link cost changes:

- node detects local link cost change
- updates routing info, recalculates distance vector
- if DV changes, notify neighbors


At time $t_{0}, y$ detects the link-cost change, updates its DV,
"good
news
travels fast" and informs its neighbors.

At time $t_{1}, z$ receives the update from $y$ and updates its table. It computes a new least cost to $x$ and sends its neighbors its DV.

At time $t_{2}, y$ receives z's update and updates its distance table. $y$ 's least costs do not change and hence $y$ does not send any message to z .

## Distance Vector: link cost changes

## Link cost changes:

- good news travels fast
- bad news travels slowly -
"count to infinity" problem!
- 44 iterations before algorithm stabilizes: see text



## Poisoned reverse:

- If $Z$ routes through $Y$ to get to $X$ :
- $Z$ tells $Y$ its ( $Z$ 's) distance to $X$ is infinite (so $Y$ won't route to $X$ via $Z$ )
- will this completely solve count to infinity problem?


## The DV Convergence Problem

- Before the link cost changes, costs are:

$$
\text { , } D y(x)=4, D y(z)=I, D z(y)=I, D z(x)=5
$$

- What does $y$ see as the shortest route to $x$ when $c(y, x)=60$ ?

$$
\begin{aligned}
D y(x) & =\min \{c(y, x)+D x(x), c(y, z)+D z(x)\} \\
& =\min \{60+0,1+5\}=6
\end{aligned}
$$

- What happens at node $z$ after this?
- $\quad \operatorname{Dy}(\mathrm{z})=\min \{c(\mathrm{z}, \mathrm{x})+\mathrm{Dx}(\mathrm{x}), \mathrm{c}(\mathrm{z}, \mathrm{y})+\mathrm{Dy}(\mathrm{x})\}$

$$
=\min \{50+0,1+6\}=7
$$

- Round and round it goes (44 times, to be exact)


## Comparison of LS and DV algorithms

## Message complexity

- LS: with $n$ nodes, E links, $\mathrm{O}(\mathrm{nE})$ msgs sent
- DV: exchange between neighbors only
- convergence time varies


## Speed of Convergence

- $\quad \mathrm{LS}: \mathrm{O}\left(\mathrm{n}^{2}\right)$ algorithm requires $\mathrm{O}(\mathrm{nE}) \mathrm{msgs}$
- may have oscillations
- DV: convergence time varies
- may be routing loops
- count-to-infinity problem


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## Hierarchical Routing

Our routing study thus far - idealization

- all routers identical
- network "flat"
... not true in practice
scale: with 600 million destinations:
- can't store all dest's in routing tables!
- routing table exchange would swamp links!


## administrative autonomy

- Internet = network of networks
- each network admin may want to control routing in its own network


## Hierarchical Routing

- aggregate routers into regions,"autonomous
systems" (AS)
- routers in same AS run same routing protocol
- "intra-AS" routing protocol
- routers in different AS can run different intra-AS routing protocol


## Gateway router

- Direct link to router in another AS


## Interconnected ASes



- Forwarding table is configured by both intraand inter-AS routing algorithm
- Intra-AS sets entries for internal dests
- Inter-AS \& Intra-As sets entries for external dests


## Inter-AS tasks

- Suppose router in ASI receives datagram for which the dest is outside of ASI
- Router should forward packet towards one of the gateway routers, but which one?


## ASI needs:

I. to learn which dests are reachable through AS2 and which through AS3
2. to propagate this reachability info to all routers in ASI

Job of inter-AS routing!


## Example: Setting forwarding table in router Id

- Suppose AS1 learns (via inter-AS protocol) that subnet x is reachable via AS3 (gateway 1c) but not via AS2.
- Inter-AS protocol propagates reachability info to all internal routers.
- Router 1d determines from intra-AS routing info that its interface $I$ is on the least cost path to 1c.
- Puts in forwarding table entry ( $\mathrm{x}, \mathrm{I}$ ).



## Example: Choosing among multiple ASes

- Now suppose AS 1 learns from the inter-AS protocol that subnet $x$ is reachable from AS 3 and from AS2.
- To configure forwarding table, router 1d must determine towards which gateway it should forward packets for dest x.
- This is also the job on inter-AS routing protocol!



## Example: Choosing among multiple ASes

- Now suppose ASI learns from the inter-AS protocol that subnet $X$ is reachable from AS3 and from AS2.
- To configure forwarding table, router Id must determine towards which gateway it should forward packets for dest $x$.
- This is also the job on inter-AS routing protocol!
- Hot potato routing: send packet towards closest of two routers.

Learn from inter-AS protocol that subnet $x$ is reachable via multiple gateways

$\rightarrow$| Use routing info <br> from intra-AS <br> protocol to determine <br> costs of least-cost <br> paths to each <br> of the gateways |
| :---: |$\rightarrow$| Hot potato routing: |
| :---: |
| Choose the gateway |
| that has the |
| smallest least cost |

\(\xrightarrow[\begin{array}{c}Determine from <br>
forwarding table the <br>
interface I that leads <br>
to least-cost gateway. <br>
Enter(x, I) in <br>

forwarding table\end{array}]{\)| .  |
| :---: |$|}$

## Next Time

- Read Sections 4.6 and 4.7
- Internet Routing and Multicast
- Project 2 - Due next Tuesday

